Solución Taller N°10 Álgebra (IME006)
Ingenierías Civiles

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Problemas.

1. (2 puntos) Sean $A, B \in \mathcal{M}_n(\mathbb{K})$ tal que $|A| = \sqrt{2}$ y $|B| = \dfrac{1}{2}$. Determine:

   a) $|AB|$ b) $|B^{-1}|$ c) $\left|(AB^{-1})^t\right|$ d) $\left|(BA^{-1})^{-1}\right|$

2. (4 puntos) Muestre que:

$$\begin{vmatrix} 1 & a_1 & a_1^2 & a_1^3 \\ 1 & a_2 & a_2^2 & a_2^3 \\ 1 & a_3 & a_3^2 & a_3^3 \\ 1 & a_4 & a_4^2 & a_4^3 \end{vmatrix} = \prod_{i>j} (a_i - a_j).$$

Solución.

1. a) $|AB| = |A| \cdot |B| = \sqrt{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{2}$.

   b) $|B^{-1}| = |B|^{-1} = \left(\frac{1}{2}\right)^{-1} = 2$.

   c) $\left|(AB^{-1})^t\right| = |(AB^{-1})| = |A| \cdot |B^{-1}| = |A| \cdot |B|^{-1} = \sqrt{2} \cdot \left(\frac{1}{2}\right)^{-1} = 2\sqrt{2}$.

   d) $\left|(BA^{-1})^{-1}\right| = |BA^{-1}|^{-1} = (|B| \cdot |A^{-1}|)^{-1} = \left(|B| \cdot |A|^{-1}\right)^{-1} = |B|^{-1} \cdot |A| = \left(\frac{1}{2}\right)^{-1} \cdot \sqrt{2} = 2\sqrt{2}$.

2. En efecto,

$$\begin{vmatrix} 1 & a_1 & a_1^2 & a_1^3 \\ 1 & a_2 & a_2^2 & a_2^3 \\ 1 & a_3 & a_3^2 & a_3^3 \\ 1 & a_4 & a_4^2 & a_4^3 \end{vmatrix} \overset{\substack{L_{21}(-1) \\ L_{31}(-1)}}{\underset{L_{41}(-1)}{=}} \begin{vmatrix} 1 & a_1 & a_1^2 & a_1^3 \\ 0 & a_2 - a_1 & a_2^2 - a_1^2 & a_2^3 - a_1^3 \\ 0 & a_3 - a_1 & a_3^2 - a_1^2 & a_3^3 - a_1^3 \\ 0 & a_4 - a_1 & a_4^2 - a_1^2 & a_4^3 - a_1^3 \end{vmatrix}$$

$$
\begin{aligned}
&= (a_2-a_1)(a_3-a_1)(a_4-a_1)\begin{vmatrix} 1 & a_1 & a_1^2 & a_1^3 \\ 0 & 1 & a_2+a_1 & a_2^2+a_1a_2+a_1^2 \\ 0 & 1 & a_3+a_1 & a_3^2+a_1a_3+a_1^2 \\ 0 & 1 & a_4+a_1 & a_4^2+a_1a_4+a_1^2 \end{vmatrix} \\
&= (a_2-a_1)(a_3-a_1)(a_4-a_1)\begin{vmatrix} 1 & a_2+a_1 & a_2^2+a_1a_2+a_1^2 \\ 1 & a_3+a_1 & a_3^2+a_1a_3+a_1^2 \\ 1 & a_4+a_1 & a_4^2+a_1a_4+a_1^2 \end{vmatrix} \\
&\underset{L_{31}(-1)}{\overset{L_{21}(-1)}{=}} (a_2-a_1)(a_3-a_1)(a_4-a_1)\begin{vmatrix} 1 & a_2+a_1 & a_2^2+a_1a_2+a_1^2 \\ 0 & a_3-a_2 & a_3^2-a_2^2+a_1(a_3-a_2) \\ 0 & a_4-a_2 & a_4^2-a_2^2+a_1(a_4-a_2) \end{vmatrix} \\
&= (a_2-a_1)(a_3-a_1)(a_4-a_1)(a_3-a_2)(a_4-a_2)\begin{vmatrix} 1 & a_3+a_2+a_1 \\ 1 & a_4+a_2+a_1 \end{vmatrix} \\
&\overset{L_{21}(-1)}{=} (a_2-a_1)(a_3-a_1)(a_4-a_1)(a_3-a_2)(a_4-a_2)\begin{vmatrix} 1 & a_3+a_2+a_1 \\ 0 & a_4-a_3 \end{vmatrix} \\
&= (a_2-a_1)(a_3-a_1)(a_4-a_1)(a_3-a_2)(a_4-a_2)(a_4-a_3) \\
&= \prod_{i>j}(a_i-a_j).
\end{aligned}
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